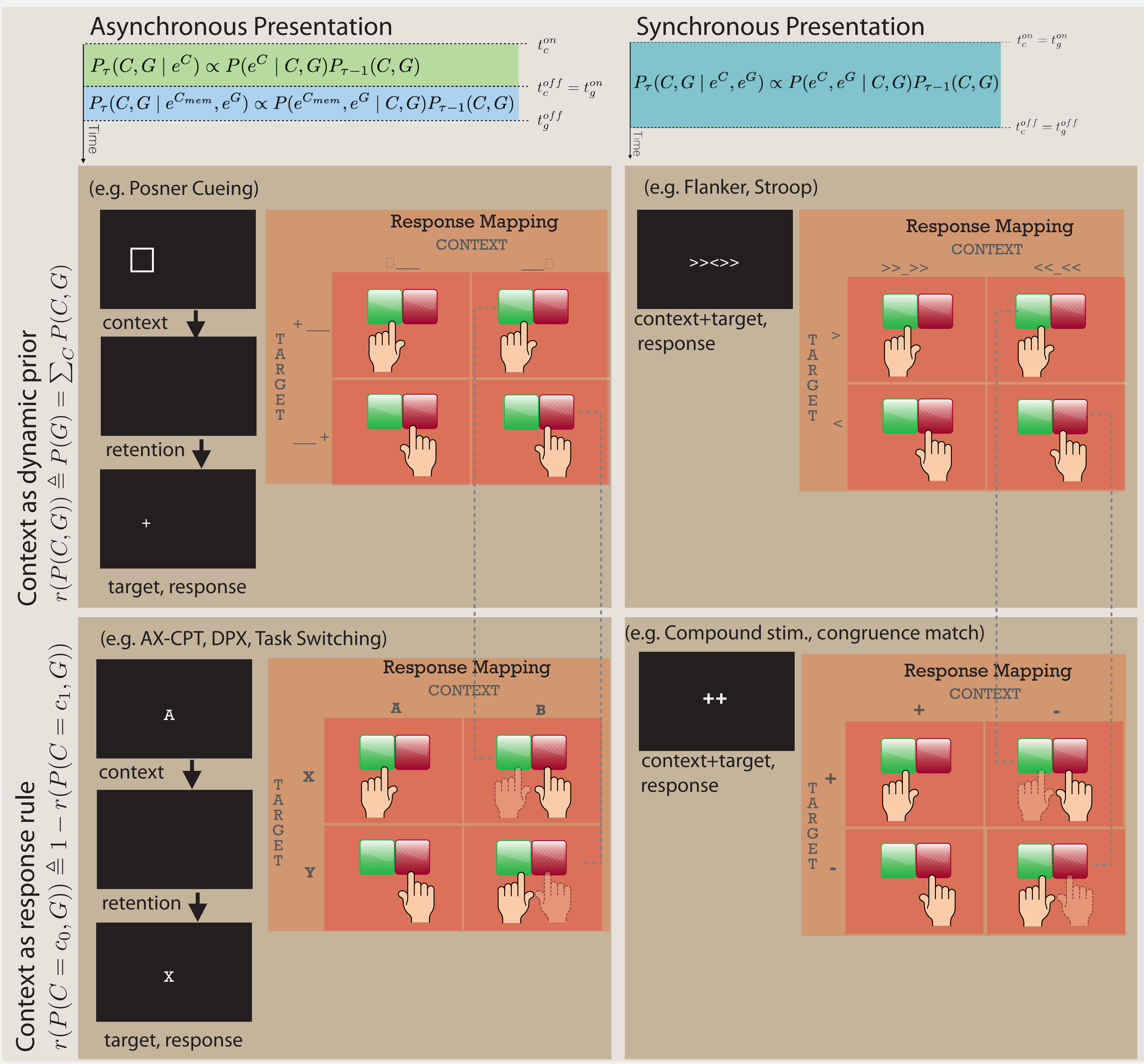
# A Theory of Decision Making Under Dynamic Context


Michael Shvartsman[1] Vaibhav Srivastava[2] Jonathan D. Cohen[1]

[1]Princeton Neuroscience Institute [2]Department of Mechanical and Aerospace Engineering
Princeton University


## (1) Decision making as sequential Bayesian inference

**How do we combine information to make decisions?** Assumption: infer a target stimulus $G$ and context stimulus $C$ from noisy evidence $e^G, e^C$; $r(\cdot)$ defines mapping from posterior to a probability of response. Respond based on fixed threshold on response probability.




## Background: from Bayesian inference to drift and diffusion

At time $\tau$, infer identity of $g \in \mathcal{G}, |\mathcal{G}| = n$ drawn from $G$ based on evidence $e^G_\tau$:

$$P_\tau(G = g \mid e^G_\tau) \propto P(e^G_\tau \mid G = g) P_{\tau-1}(G = g \mid e_{\tau-1})$$

Decide when $\max_g(G = g) > \theta$, pick option $\arg\max_g(G = g) > \theta$.

If there are only two hypotheses:

$$\log Z^\tau_g \triangleq \log \frac{P(e^G_\tau \mid G = g_0) P_{\tau-1}(G = g_0)}{P(e^G_\tau \mid G = g_1) P_{\tau-1}(G = g_1)} = \log \frac{P_0(G = g_0)}{P_0(G = g_1)} + \sum_{t=1}^{\tau} \log \frac{P(e^G_t \mid G = g_0)}{P(e^G_t \mid G = g_1)}$$

Define $\delta t \triangleq$ duration of a single update, and look at the limit as $\delta t \to 0$:

$$dz_g = z^0_g + a_g dt + b_g dW$$

$$z^0_g = \log \frac{P_0(G = g_0)}{P_0(G = g_1)}$$

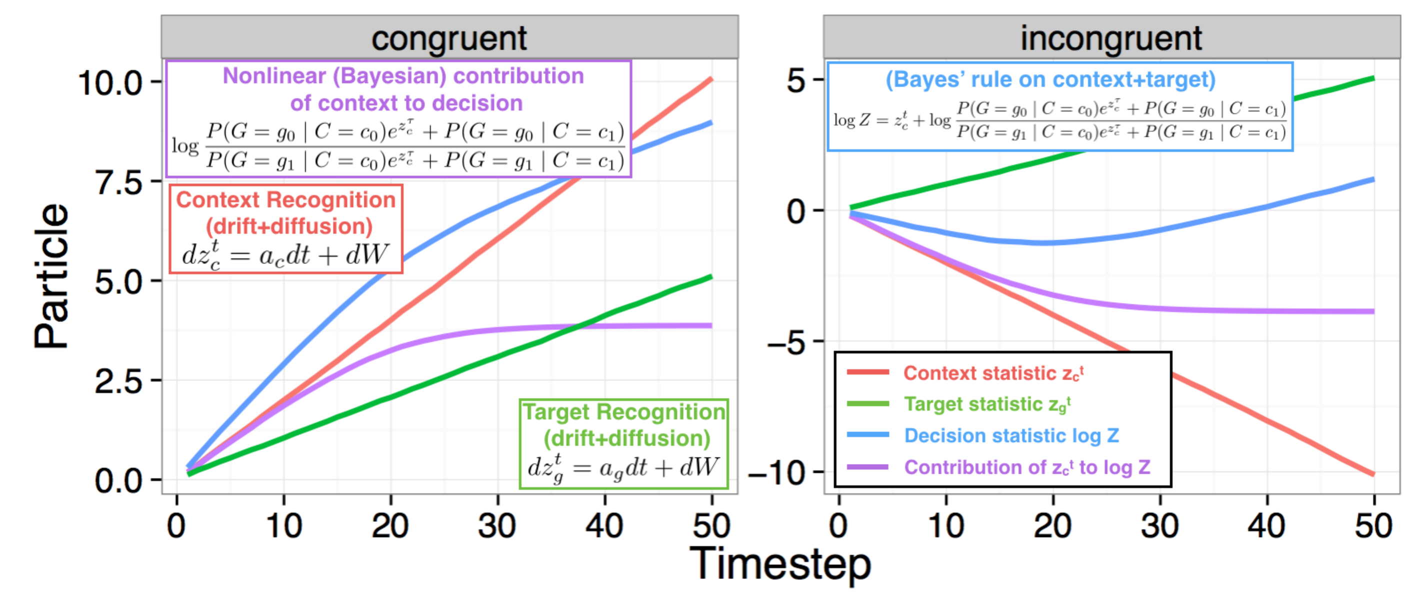
$$a_g = \mathbb{E}\left[\log \frac{P(e^G_t \mid G = g_0)}{P(e^G_t \mid G = g_1)}\right]$$

$$b^2_g = \mathrm{Var}\left[\log \frac{P(e^G_t \mid G = g_0)}{P(e^G_t \mid G = g_1)}\right]$$

In discrete form: Wald's Sequential Probability Ratio Test (SPRT): optimal test for two hypotheses (Wald 1947). In continuous form: Ratcliff's Diffusion Decision Model (DDM; Ratcliff 1978). In either form, emerging consensus theory of the behavioral/neural dynamics of simple decisions (Edwards, 1965; Gold & Shadlen 2002; Kira et al. 2015).

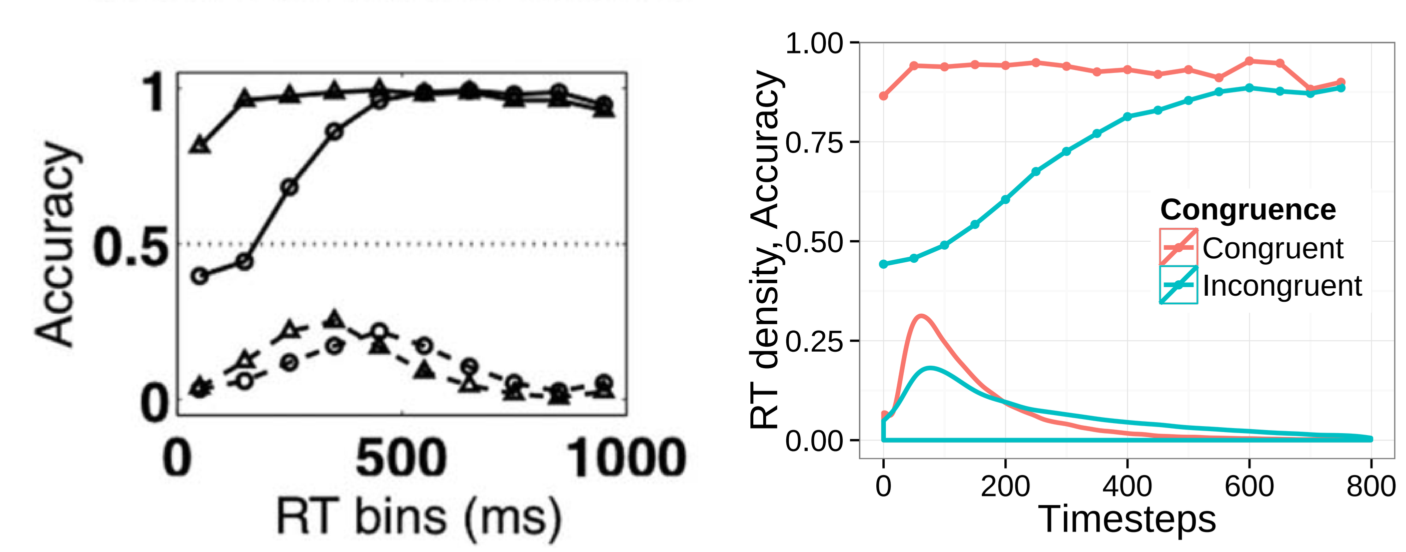

**References**: Braver (2012). TiCS, 16(2), 106–13; Edwards (1965) J. Math. Psych. 2, 312-329; Gold & Shadlen (2002). Neuron, 36(2), 299-308; Kira, Yang, & Shadlen (2015). Neuron, 85(4), 861–873; Ratcliff (1978) Psych. Rev., 85(2), 59–108; Servan-Schreiber, Bruno, Carter, & Cohen (1998). Bio. Psych., 43(10), 713–722; Wald, & Wolfowitz (1948). Annals of Math. Stat., 19(3), 326–339; Yu, Dayan, & Cohen (2009). JEP:HPP, 35(3), 700–717


## (2) Generalizes previous model of Flanker task



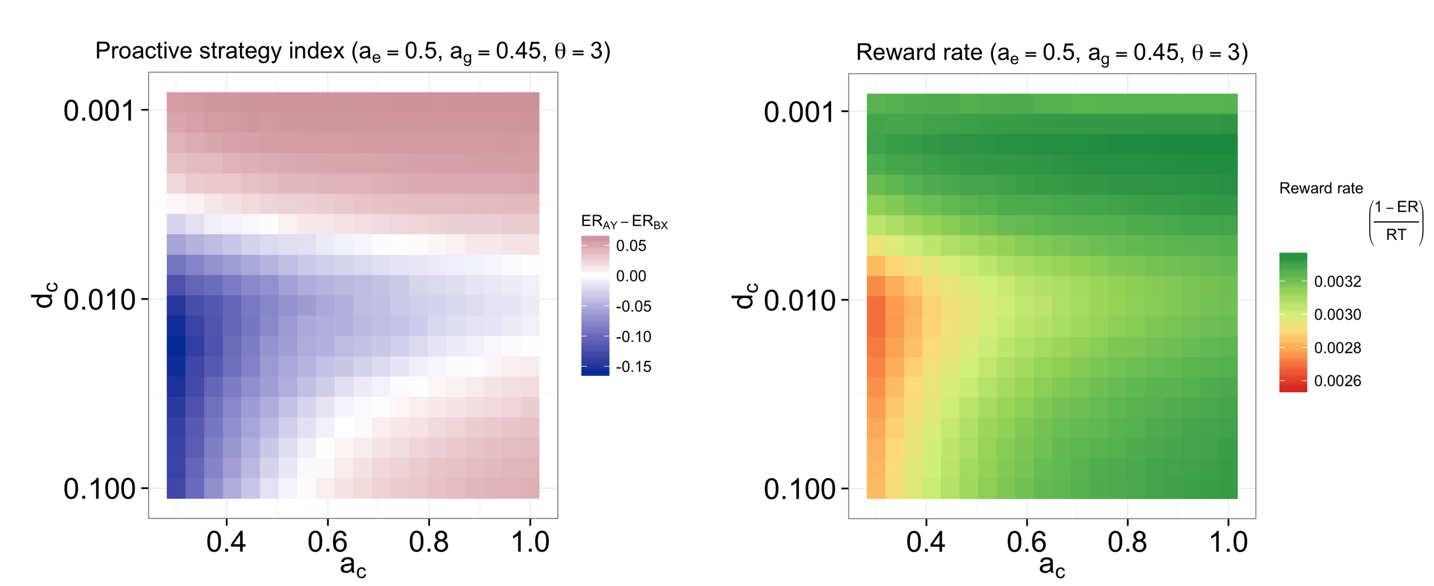

Average dynamics of model for congruent and incongruent Flanker trial. Notationally equivalent to a model by Yu, Dayan & Cohen 2009 after relabeling posterior events.

**From Servan-Shreiber et al, 1998**

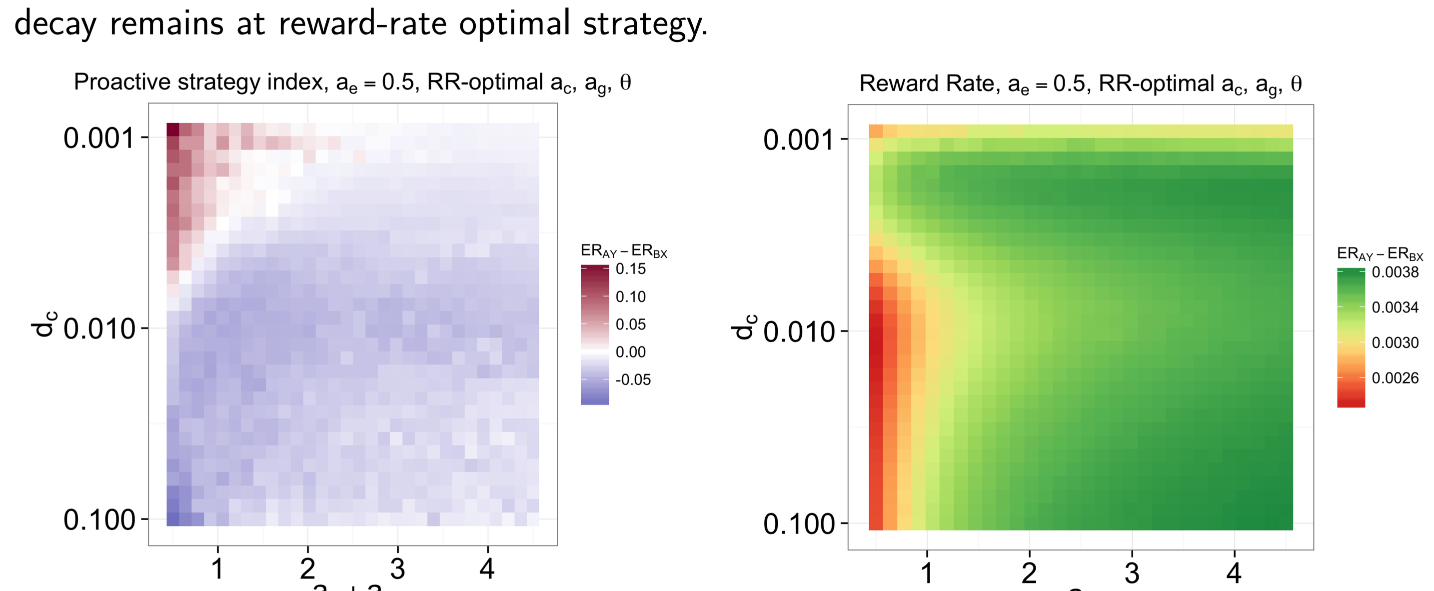



**Left**: the Gratton effect (below-chance performance on early responses). **Right**: model recovers same pattern (generalization of Yu et al. model).

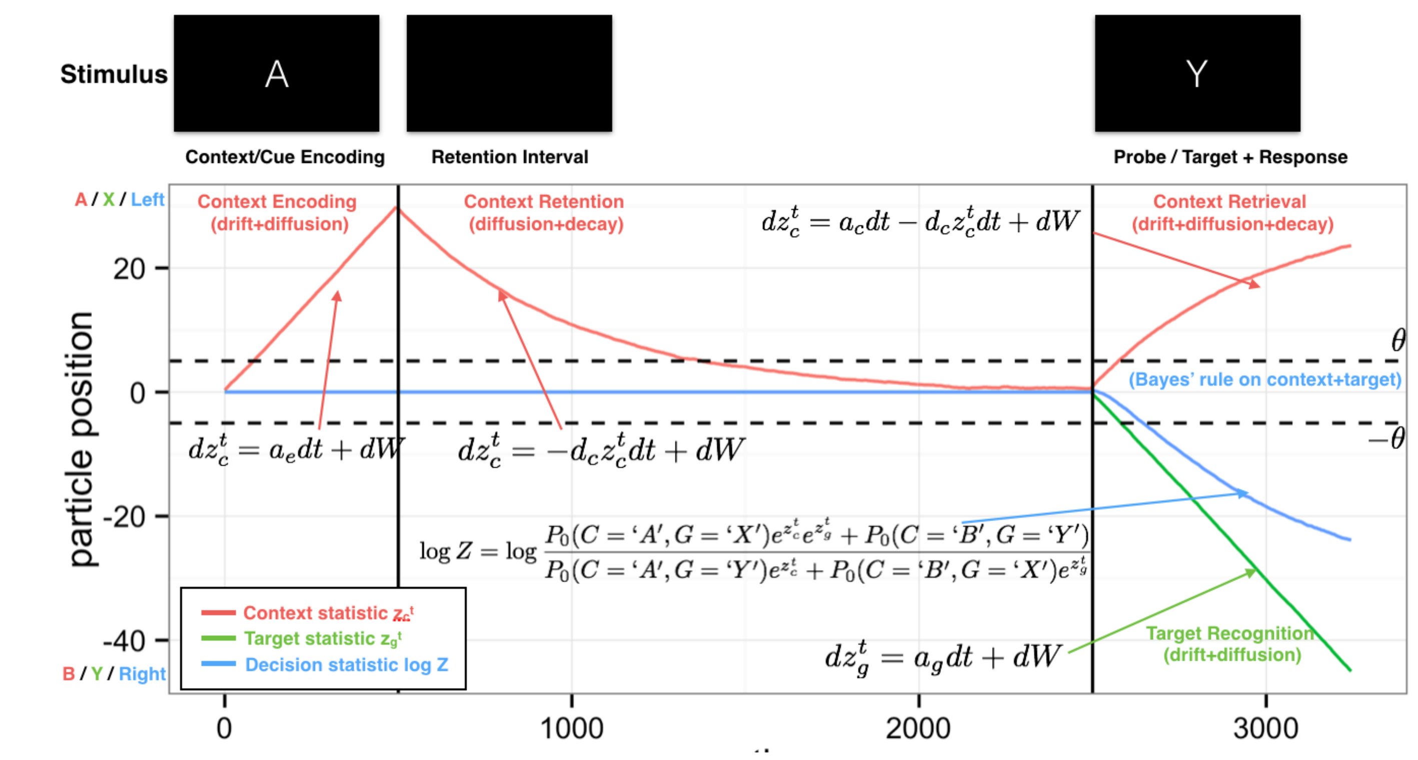
## (4) Bounded optimal control in AX-CPT



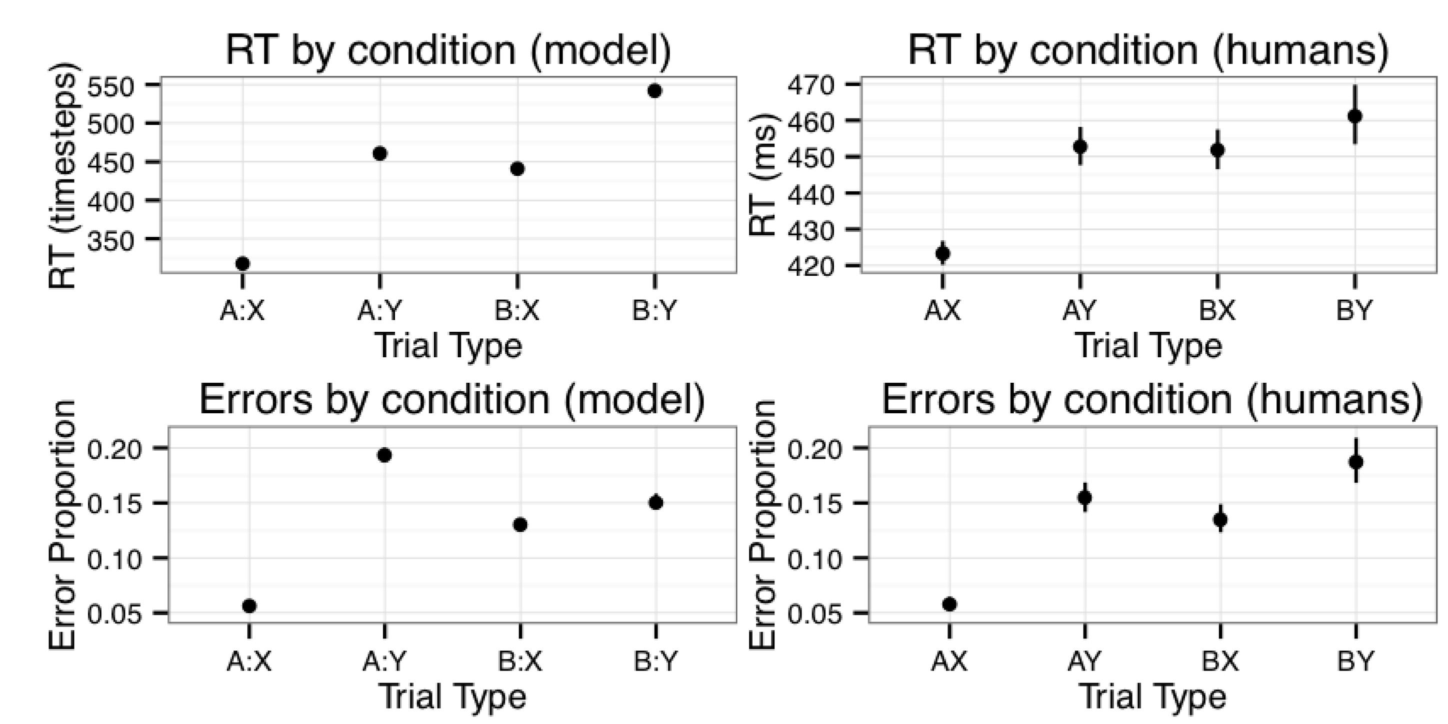

**Above left**: model recovers behavioral signature of both "proactive" and "reactive" control in AX-CPT (Braver 2012). **Above right**: increased reward rate with higher decay suggests a rational benefit of memory decay. **Below left**: proactive/reactive behavior index at reward-rate optimal setting of threshold, target and retrieval drift ($a_c, a_g$). Proactive behavior emerges when decay is low but so is total capacity. **Below right**: beneficial role of decay remains at reward-rate optimal strategy.



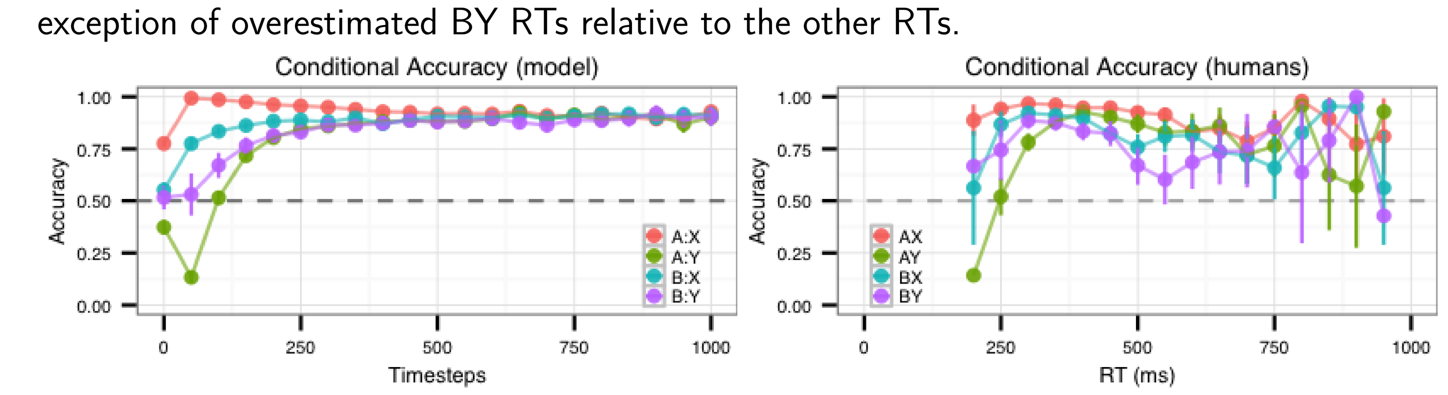

## (3) Recovers AX-CPT Behavior under same parameters




Average dynamics of model for AY trial of AX-CPT.




**Above**: RT and error rate predictions (left) of model and comparison to humans (right) under same parameters as flanker task (taken from Yu et al. 2009); **Below**: accuray by RT bin for the model (left) and humans (right). The model recovers the full pattern, with exception of overestimated BY RTs relative to the other RTs.

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## Contact Information


- Web: http://www.princeton.edu/~ms44
- Email: ms44@princeton.edu
- Code: https://github.com/mshvartsman/cddm